7-7 Base e and Natural Logarithms

Write an equivalent exponential or logarithmic function.

1. $e^x = 30$

SOLUTION:

$$\begin{aligned} e^x &= 30 \\ \log_e 30 &= x \\ \ln 30 &= x \end{aligned}$$

2. $\ln x = 42$

SOLUTION:

$$\begin{aligned} &\ln x = 42 \\ &\log_e x = 42 \\ &e^{42} = x \end{aligned}$$

3. $e^3 = x$

SOLUTION:

$$\begin{aligned} e^3 &= x \\ \log_e x &= 3 \\ \ln x &= 3 \end{aligned}$$

4. $\ln 18 = x$

SOLUTION:

$$\begin{aligned} \ln 18 &= x \\ \log_e 18 &= x \\ e^x &= 18 \end{aligned}$$

Write each as a single logarithm.

5. $3 \ln 2 + 2 \ln 4$

SOLUTION:

$$\begin{aligned} 3\ln 2 + 2\ln 4 &= \ln 2^3 + \ln 4^2 \\ &= \ln 8 + \ln 16 \\ &= \ln(8 \times 16) \\ &= \ln 128 \\ &= \ln 2^7 \\ &= 7 \ln 2 \end{aligned}$$

6. $5 \ln 3 - 2 \ln 9$

SOLUTION:

$$\begin{aligned} 5\ln 3 - 2\ln 9 &= \ln 3^5 - \ln 9^2 \\ &= \ln 243 - \ln 81 \\ &= \ln\left(\frac{243}{81}\right) \\ &= \ln 3 \end{aligned}$$

7. $3 \ln 6 + 2 \ln 9$

SOLUTION:

$$\begin{aligned} 3\ln 6 + 2\ln 9 &= \ln 6^3 + \ln 9^2 \\ &= \ln 216 + \ln 81 \\ &= \ln(216 \times 81) \\ &= \ln 17496 \end{aligned}$$

8. $3 \ln 5 + 4 \ln x$

SOLUTION:

$$\begin{aligned} 3\ln 5 + 4\ln x &= \ln 5^3 + \ln x^4 \\ &= \ln 125 + \ln x^4 \\ &= \ln 125x^4 \end{aligned}$$

Solve each equation. Round to the nearest ten-thousandth.

9. $5e^x - 24 = 16$

SOLUTION:

$$\begin{aligned} 5e^x - 24 &= 16 \\ 5e^x &= 40 \\ e^x &= 8 \\ \ln e^x &= \ln 8 \\ x &= \ln 8 \\ &\approx 2.0794 \end{aligned}$$

10. $-3e^x + 9 = 4$

SOLUTION:

$$\begin{aligned}
-3e^x + 9 &= 4 \\
-3e^x &= -5 \\
e^x &= \frac{-5}{-3} \\
e^x &= \frac{5}{3} \\
\ln e^x &= \ln \frac{5}{3} \\
x &= \ln \frac{5}{3} \\
&\approx 0.5108
\end{aligned}$$

11. $3e^{-3x} + 4 = 6$

SOLUTION:

$$\begin{aligned}
3e^{-3x} + 4 &= 6 \\
3e^{-3x} &= 2 \\
e^{-3x} &= \frac{2}{3} \\
\ln e^{-3x} &= \ln \frac{2}{3} \\
-3x &= \ln \frac{2}{3} \\
x &= \frac{\ln \frac{2}{3}}{-3} \\
&\approx 0.1352
\end{aligned}$$

12. $2e^{-x} - 3 = 8$

SOLUTION:

$$\begin{aligned}
2e^{-x} - 3 &= 8 \\
2e^{-x} &= 11 \\
e^{-x} &= \frac{11}{2} \\
\ln e^{-x} &= \ln \frac{11}{2} \\
-x &= \ln \frac{11}{2} \\
x &= -\ln \frac{11}{2} \\
&\approx -1.7047
\end{aligned}$$

Solve each equation or inequality. Round to the nearest ten-thousandth.

13. $\ln 3x = 8$

SOLUTION:

$$\begin{aligned}\ln 3x &= 8\\ 3x &= e^8\\ x &= \frac{e^8}{3}\\ &\approx 993.6527\end{aligned}$$

The solution is 999.36527.

14. $-4 \ln 2x = -26$

SOLUTION:

$$\begin{aligned}-4\ln 2x &= -26\\ \ln 2x &= \frac{-26}{-4}\\ \ln 2x &= \frac{13}{2}\\ 2x &= e^{6.5}\\ x &= \frac{e^{6.5}}{2}\\ &\approx 332.5708\end{aligned}$$

The solution is 332.5708.

15. $\ln (x + 5)^2 < 6$

SOLUTION:

$$\begin{aligned}\ln(x+5)^2 &< 6\\ e^{\ln(x+5)^2} &< e^6\\ (x+5)^2 &< e^6\\ \sqrt{(x+5)^2} &< \sqrt{e^6}\\ x+5 &< \pm e^6\end{aligned}$$

$$\begin{aligned}&-e^6 < x+5 < e^6\\ &-e^6 - 5 < x < e^6 - 5\\ &-25.0855 < x < 15.0855\end{aligned}$$

The solution region is $\{x \mid -25.0855 < x < 15.0855 \quad x \neq -5\}$.

16. $\ln(x-2)^3 > 15$

SOLUTION:

$$\begin{aligned}\ln(x-2)^3 &> 15\\ 3\ln(x-2) &> 15\\ \ln(x-2) &> 5\\ x-2 &> e^5\\ x &> e^5+2\\ x &> 150.4132\end{aligned}$$

The solution region is $\{x \mid x > 150.4132\}$.

17. $e^x > 29$

SOLUTION:

$$\begin{aligned}e^x &> 29\\ \ln e^x &> \ln 29\\ x &> \ln 29\\ x &> 3.3673\end{aligned}$$

The solution region is $\{x \mid x > 3.3673\}$.

18. $5 + e^{-x} > 14$

SOLUTION:

$$\begin{aligned}5+e^{-x} &> 14\\ e^{-x} &> 9\\ \ln e^{-x} &> \ln 9\\ -x &> \ln 9\\ x &< -\ln 9\\ x &< -2.1972\end{aligned}$$

The solution region is $\{x \mid x < -2.1972\}$.

19. **SCIENCE** A virus is spreading through a computer network according to the formula $v(t) = 30e^{0.1t}$, where v is the number of computers infected and t is the time in minutes. How long will it take the virus to infect 10,000 computers?

SOLUTION:

Substitute 10,000 for $v(t)$ and solve for t.

$$\begin{aligned} 30e^{0.1t} &= 10000 \\ e^{0.1t} &= \frac{10000}{30} \\ 0.1t &= \ln\frac{10000}{30} \\ t &= \frac{1}{0.1}\ln\frac{10000}{30} \\ &\approx 58 \end{aligned}$$

The virus will take about 58 min to infect 10,000 computers.

Write an equivalent exponential or logarithmic function.

20. $e^{-x} = 8$

SOLUTION:

$$\begin{aligned} e^{-x} &= 8 \\ \log_e 8 &= -x \\ \ln 8 &= -x \end{aligned}$$

21. $e^{-5x} = 0.1$

SOLUTION:

$$\begin{aligned} e^{-5x} &= 0.1 \\ \log_e 0.1 &= -5x \\ \ln 0.1 &= -5x \end{aligned}$$

22. $\ln 0.25 = x$

SOLUTION:

$$\begin{aligned} \ln 0.25 &= x \\ \log_e 0.25 &= x \\ 0.25 &= e^x \end{aligned}$$

23. $\ln 5.4 = x$

SOLUTION:

$$\begin{aligned} \ln 5.4 &= x \\ \log_e 5.4 &= x \\ 5.4 &= e^x \end{aligned}$$

24. $e^{x-3} = 2$

SOLUTION:

$$\begin{aligned} e^{x-3} &= 2 \\ \log_e 2 &= x - 3 \\ \ln 2 &= x - 3 \end{aligned}$$

25. $\ln(x + 4) = 36$

SOLUTION:

$$\begin{aligned} \ln(x+4) &= 36 \\ \log_e(x+4) &= 36 \\ x + 4 &= e^{36} \end{aligned}$$

26. $e^{-2} = x^6$

SOLUTION:

$$\begin{aligned} e^{-2} &= x^6 \\ \log_e x^6 &= -2 \\ \ln x^6 &= -2 \\ 6\ln x &= -2 \end{aligned}$$

27. $\ln e^x = 7$

SOLUTION:

$$\begin{aligned} \ln e^x &= 7 \\ \log_e e^x &= 7 \\ e^7 &= e^x \end{aligned}$$

Write each as a single logarithm.

28. $\ln 125 - 2\ln 5$

SOLUTION:

$$\begin{aligned} \ln 125 - 2\ln 5 &= \ln 125 - \ln 5^2 \\ &= \ln 125 - \ln 25 \\ &= \ln\frac{125}{25} \\ &= \ln 5 \end{aligned}$$

7-7 Base e and Natural Logarithms

29. $3 \ln 10 + 2 \ln 100$

SOLUTION:

$$\begin{aligned} 3\ln 10 + 2\ln 100 &= \ln 10^3 + \ln 100^2 \\ &= \ln\left(10^3 \times 100^2\right) \\ &= \ln 10^7 \\ &= 7\ln 10 \end{aligned}$$

30. $4\ln\frac{1}{3} - 6\ln\frac{1}{9}$

SOLUTION:

$$\begin{aligned} 4\ln\frac{1}{3} - 6\ln\frac{1}{9} &= \ln\left(\frac{1}{3}\right)^4 - \ln\left(\frac{1}{9}\right)^6 \\ &= \ln\frac{1}{3^4} - \ln\frac{1}{9^6} \\ &= \ln\left(\frac{\frac{1}{3^4}}{\frac{1}{9^6}}\right) \\ &= \ln\frac{9^6}{3^4} \\ &= \ln\frac{3^{12}}{3^4} \\ &= \ln 3^8 \\ &= 8\ln 3 \\ &= -8\ln\frac{1}{3} \end{aligned}$$

31. $7\ln\frac{1}{2} + 5\ln 2$

SOLUTION:

$$\begin{aligned} 7\ln\frac{1}{2} + 5\ln 2 &= \ln\left(\frac{1}{2}\right)^7 + \ln 2^5 \\ &= \ln\frac{1}{2^7} + \ln 2^5 \\ &= \ln\left(\frac{1}{2^7}\cdot 2^5\right) \\ &= \ln\frac{1}{2^2} \\ &= \ln 2^{-2} \\ &= -2\ln 2 \end{aligned}$$

32. $8 \ln x - 4 \ln 5$

SOLUTION:

$$\begin{aligned} 8 \ln x - 4 \ln 5 &= \ln x^8 - \ln 5^4 \\ &= \ln \frac{x^8}{5^4} \\ &= \ln \frac{x^8}{625} \end{aligned}$$

33. $3 \ln x^2 + 4 \ln 3$

SOLUTION:

$$\begin{aligned} 3 \ln x^2 + 4 \ln 3 &= \ln \left(x^2\right)^3 + \ln 3^4 \\ &= \ln x^6 + \ln 81 \\ &= \ln 81x^6 \end{aligned}$$

Solve each equation. Round to the nearest ten-thousandth.

34. $6e^x - 3 = 35$

SOLUTION:

$$\begin{aligned} 6e^x - 3 &= 35 \\ 6e^x &= 38 \\ e^x &= \frac{38}{6} \\ x &= \ln \frac{38}{6} \\ &\approx 1.8458 \end{aligned}$$

The solution is 1.8458.

35. $4e^x + 2 = 180$

SOLUTION:

$$\begin{aligned} 4e^x + 2 &= 180 \\ 4e^x &= 178 \\ e^x &= \frac{178}{4} \\ x &= \ln \frac{178}{4} \\ &\approx 3.7955 \end{aligned}$$

The solution is 3.7955.

36. $3e^{2x} - 5 = -4$

SOLUTION:

$$\begin{aligned} 3e^{2x} - 5 &= -4 \\ 3e^{2x} &= 1 \\ e^{2x} &= \frac{1}{3} \\ 2x &= \ln\frac{1}{3} \\ x &= \frac{\ln\frac{1}{3}}{2} \\ &\approx -0.5493 \end{aligned}$$

The solution is –0.5493.

37. $-2e^{3x} + 19 = 3$

SOLUTION:

$$\begin{aligned} -2e^{3x} + 19 &= 3 \\ -2e^{3x} &= -16 \\ e^{3x} &= 8 \\ \ln e^{3x} &= \ln 8 \\ 3x &= \ln 8 \\ x &= \frac{\ln 8}{3} \\ x &\approx 0.6931 \end{aligned}$$

The solution is 0.6931.

38. $6e^{4x} + 7 = 4$

SOLUTION:

$$\begin{aligned} 6e^{4x} + 7 &= 4 \\ 6e^{4x} &= -3 \\ e^{4x} &= -\frac{1}{2} \\ \ln e^{4x} &= \ln\left(-\frac{1}{2}\right) \end{aligned}$$

Logarithm is not defined for negative values.
Therefore, there is no solution.

39. $-4e^{-x} + 9 = 2$

SOLUTION:

$$
\begin{aligned}
-4e^{-x} + 9 &= 2 \\
-4e^{-x} &= -7 \\
e^{-x} &= \frac{7}{4} \\
\ln e^{-x} &= \ln \frac{7}{4} \\
-x &= \ln \frac{7}{4} \\
x &= -\ln \frac{7}{4} \\
x &\approx -0.5596
\end{aligned}
$$

The solution is –0.5596

40. **CCSS SENSE-MAKING** The value of a certain car depreciates according to $v(t) = 18500e^{-0.186t}$, where t is the number of years after the car is purchased new.
a. What will the car be worth in 18 months?
b. When will the car be worth half of its original value?
c. When will the car be worth less than $1000?

SOLUTION:

a. 18 months is equal to 1.5 years.
Substitute 1.5 for t and evaluate.

$$\begin{aligned} v(t) &= 18500e^{-0.186(1.5)} \\ &= 18500e^{-0.279} \\ &\approx 13996 \end{aligned}$$

The car will be worth about 13,996 in 18 months.

b. Substitute 9250 for $v(t)$ and solve for t.

$$\begin{aligned} 9250 &= 18500e^{-0.186t} \\ \frac{9250}{18500} &= e^{-0.186t} \\ e^{-0.186t} &= \frac{1}{2} \\ -0.186t &= \ln\frac{1}{2} \\ t &= -\frac{1}{0.186}\ln\frac{1}{2} \\ &\approx 3.73 \end{aligned}$$

The car will be worth half of its original value in about 3.73 years.

c. Substitute 1,000 for $v(t)$ and solve for t.

$$\begin{aligned} 1000 &= 18500e^{-0.186t} \\ e^{-0.186t} &= \frac{1000}{18500} \\ -0.186t &= \ln\frac{1000}{18500} \\ t &= -\frac{1}{0.186}\ln\frac{1000}{18500} \\ &\approx 15.69 \end{aligned}$$

The car will be worth less than $1000 after 15.69 years.

Solve each inequality. Round to the nearest ten-thousandth.

41. $e^x \le 8.7$

SOLUTION:

$$\begin{aligned} e^x &\le 8.7 \\ \ln e^x &\le \ln 8.7 \\ x &\le \ln 8.7 \\ x &\le 2.1633 \end{aligned}$$

The solutions are $\{x \mid x \le 2.1633\}$.

42. $e^x \ge 42.1$

SOLUTION:

$$\begin{aligned} e^x &\ge 42.1 \\ \ln e^x &\ge \ln 42.1 \\ x &\ge \ln 42.1 \\ x &\ge 3.7400 \end{aligned}$$

The solutions are $\{x \mid x \ge 3.7400\}$.

43. $\ln (3x + 4)^3 > 10$

SOLUTION:

$$\begin{aligned} \ln(3x+4)^3 &> 10 \\ 3\ln(3x+4) &> 10 \\ \ln(3x+4) &> \frac{10}{3} \\ \log_e(3x+4) &> \frac{10}{3} \\ 3x+4 &> e^{\frac{10}{3}} \\ 3x &> e^{\frac{10}{3}} - 4 \\ x &> \frac{e^{\frac{10}{3}} - 4}{3} \\ x &> 8.0105 \end{aligned}$$

The solutions are $\{x \mid x > 8.0105\}$.

44. $4 \ln x^2 < 72$

SOLUTION:

$$\begin{aligned} 4\ln x^2 &< 72 \\ \ln x^2 &< 18 \\ e^{\ln x^2} &< e^{18} \\ x^2 &< e^{18} \\ \sqrt{x^2} &< \sqrt{e^{18}} \\ x &< \pm e^9 \\ -e^9 < x &< e^9 \\ -8103.0839 < x &< 8103.0839 \end{aligned}$$

The solutions are $\{x \mid -8103.0839 < x < 8103.0839\}$.

45. $\ln (8x^4) > 24$

SOLUTION:

$$\begin{aligned} \ln\left(8x^4\right) &> 24 \\ e^{\ln\left(8x^4\right)} &> e^{24} \\ 8x^4 &> e^{24} \\ x^4 &> \frac{e^{24}}{8} \\ \sqrt[4]{x^4} &> \sqrt[4]{\frac{e^{24}}{8}} \\ x > \frac{e^6}{\sqrt[4]{8}} \quad &or \quad x < -\frac{e^6}{\sqrt[4]{8}} \\ x > 239.8802 \quad &or \quad x < -239.8802 \end{aligned}$$

The solutions are $\{x \mid x > 239.8802 \text{ or } x < -239.8802\}$.

46. $-2[\ln(x-6)^{-1}] \le 6$

SOLUTION:

$$\begin{aligned}
-2[\ln(x-6)^{-1}] &\le 6\\
2\ln(x-6) &\le 6\\
\ln(x-6) &\le 3\\
\log_e(x-6) &\le 3\\
x-6 &\le e^3\\
x &\le e^3+6\\
x &\le 26.0855
\end{aligned}$$

Logarithms are not defined for negative values. So, the inequality is defined for $x - 6 > 0$.

Therefore, $x > 6$.
The solutions are $\{x \mid 6 < x \le 26.0855\}$.

47. **FINANCIAL LITERACY** Use the formula for continuously compounded interest.
a. If you deposited $800 in an account paying 4.5% interest compounded continuously, how much money would be in the account in 5 years?
b. How long would it take you to double your money?
c. If you want to double your money in 9 years, what rate would you need?
d. If you want to open an account that pays 4.75% interest compounded continuously and have $10,000 in the account 12 years after your deposit, how much would you need to deposit?

SOLUTION:

a. Substitute 800, 0.045and 5 for P, r and t in the continuously compounded interest.

$$\begin{aligned}
A &= Pe^{rt}\\
&= 800e^{0.045(5)}\\
&= 800e^{0.225}\\
&= 1001.86
\end{aligned}$$

b. Substitute 1600, 800 and 0.045 for A, P and r in the continuously compounded interest.

$$\begin{aligned}
A &= Pe^{rt}\\
1600 &= 800e^{0.045t}\\
2 &= e^{0.045t}\\
0.045t &= \ln 2\\
t &= \frac{\ln 2}{0.045}\\
&\approx 15.4
\end{aligned}$$

c. Substitute 1600, 800 and 9 for A, P and t in the continuously compounded interest.

$$
\begin{aligned}
A &= Pe^{rt} \\
1600 &= 800e^{9r} \\
2 &= e^{9r} \\
9r &= \ln 2 \\
r &= \frac{\ln 2}{9} \\
&\approx 0.077 \\
&= 7.7\%
\end{aligned}
$$

d. Substitute 10000, 0.0475 and 12 for A, r and t in the continuously compounded interest.

$$
\begin{aligned}
A &= Pe^{rt} \\
10000 &= Pe^{0.0475(12)} \\
10000 &= Pe^{0.57} \\
P &= \frac{10000}{e^{0.57}} \\
&\approx 5655.25
\end{aligned}
$$

Write the expression as a sum or difference of logarithms or multiples of logarithms.

48. $\ln 12x^2$

SOLUTION:

$$
\begin{aligned}
\ln 12x^2 &= \ln 12 + \ln x^2 \\
&= \ln 12 + 2\ln x
\end{aligned}
$$

49. $\ln \frac{16}{125}$

SOLUTION:

$$
\begin{aligned}
\ln \frac{16}{125} &= \ln 16 - \ln 125 \\
&= \ln 4^2 - \ln 5^3 \\
&= 2\ln 4 - 3\ln 5
\end{aligned}
$$

50. $\ln \sqrt[5]{x^3}$

SOLUTION:

$$
\begin{aligned}
\ln \sqrt[5]{x^3} &= \ln\left(x^3\right)^{\frac{1}{5}} \\
&= \frac{1}{5}\ln x^3 \\
&= \frac{3}{5}\ln x
\end{aligned}
$$

51. $\ln xy^4z^{-3}$

SOLUTION:

$$\begin{aligned}\ln xy^4x^{-3} &= \ln x + \ln y^4 + \ln z^{-3}\\ &= \ln x + 4\ln y - 3\ln z\end{aligned}$$

Use the natural logarithm to solve each equation.

52. $8^x = 24$

SOLUTION:

$$\begin{aligned}8^x &= 24\\ \ln 8^x &= \ln 24\\ x\ln 8 &= \ln 24\\ x &= \frac{\ln 24}{\ln 8}\\ &\approx 1.5283\end{aligned}$$

The solution is about 1.5283.

53. $3^x = 0.4$

SOLUTION:

$$\begin{aligned}3^x &= 0.4\\ \ln 3^x &= \ln 0.4\\ x\ln 3 &= \ln 0.4\\ x &= \frac{\ln 0.4}{\ln 3}\\ x &\approx -0.8340\end{aligned}$$

The solution is about –0.8340.

54. $2^{3x} = 18$

SOLUTION:

$$\begin{aligned}2^{3x} &= 18\\ \ln 2^{3x} &= \ln 18\\ 3x\ln 2 &= \ln 18\\ 3x &= \frac{\ln 18}{\ln 2}\\ x &= \frac{1}{3}\cdot\frac{\ln 18}{\ln 2}\\ x &\approx 1.3900\end{aligned}$$

The solution is 1.3900.

55. $5^{2x} = 38$

SOLUTION:

$$\begin{aligned} 5^{2x} &= 38 \\ \ln 5^{2x} &= \ln 38 \\ 2x \ln 5 &= \ln 38 \\ 2x &= \frac{\ln 38}{\ln 5} \\ x &= \frac{1}{2} \cdot \frac{\ln 38}{\ln 5} \\ x &\approx 1.1301 \end{aligned}$$

The solution is 1.1301.

56. **CCSS MODELING** Newton's Law of Cooling, which can be used to determine how fast an object will cool in given surroundings, is represented by $T(t) = T_s + (T_0 - T_s)e^{-kt}$, where T_0 is the initial temperature of the object, T_s is the temperature of the surroundings, t is the time in minutes, and k is a constant value that depends on the type of object.

a. If a cup of coffee with an initial temperature of 180º is placed in a room with a temperature of 70º, then the coffee cools to 140º after 10 minutes, find the value of k.

b. Use this value of k to determine the temperature of the coffee after 20 minutes.

c. When will the temperature of the coffee reach 75º?

SOLUTION:

a. Substitute 180, 70, 10 and 140 for T_0, T_s, t and $T(t)$ respectively then solve for k.

$$\begin{aligned} T(t) &= T_s + (T_0 - T_s)e^{-kt} \\ 140 &= 70 + (180 - 70)e^{-10k} \\ 140 &= 70 + 110e^{-10k} \\ 70 &= 110e^{-10k} \\ \frac{7}{11} &= e^{-10k} \\ \ln \frac{7}{11} &= -10k \\ \frac{\ln \frac{7}{11}}{-10} &= k \\ k &\approx 0.045 \end{aligned}$$

The value of k is about 0.045.

b. Substitute 0.094446, 180, 70 and 20 for k, T_0, T_s and t respectively and simplify.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$
$$T(t) = 70 + (180 - 70)e^{-(0.045)(20)}$$
$$T(t) = 70 + 110e^{-0.9}$$
$$T(t) \approx 114.7$$

The temperature of the coffee after 20 minutes is about 114.7º.

c. Substitute 0.094446, 180, 70 and 75 for k, T_0, T_s and $T(t)$ respectively then solve for t.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$
$$75 = 70 + (180 - 70)e^{-0.045t}$$
$$75 = 70 + 110e^{-0.045t}$$
$$5 = 110e^{-0.045t}$$
$$\frac{1}{22} = e^{-0.045t}$$
$$\ln\frac{1}{22} = -0.045t$$
$$\frac{\ln\frac{1}{22}}{-0.045} = t$$
$$t \approx 68$$

The temperature of the coffee will reach 75º in about 68 min.

57. **MULTIPLE REPRESENTATIONS** In this problem, you will use $f(x) = e^x$ and $g(x) = \ln x$.

a. GRAPHICAL Graph both functions and their axis of symmetry, $y = x$, for $-5 \le x \le 5$. Then graph $a(x) = e^{-x}$ on the same graph.

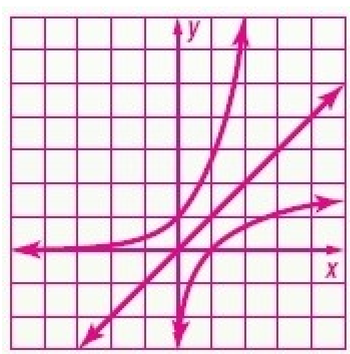
b. ANALYTICAL The graphs of $a(x)$ and $f(x)$ are reflections along which axis? What function would be a reflection of $f(x)$ along the other axis?

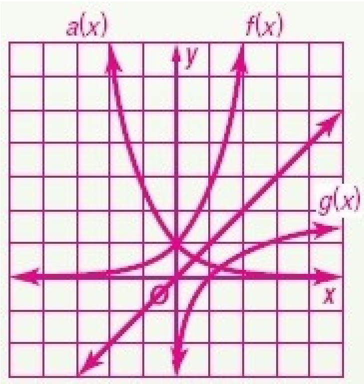
c. LOGICAL Determine the two functions that are reflections of $g(x)$. Graph these new functions.

d. VERBAL We know that $f(x)$ and $g(x)$ are inverses. Are any of the other functions that we have graphed inverses as well? Explain your reasoning.

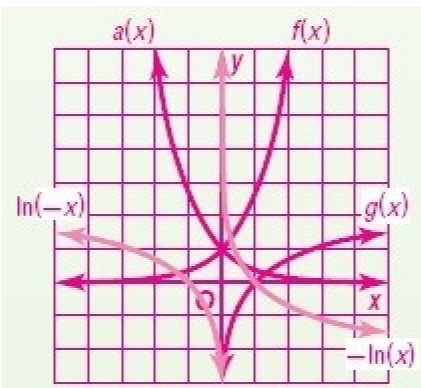
SOLUTION:

a.





b. y-axis; a(x) = $-e^x$
c. ln ($-x$) is a reflection across the y-axis. $-$ln x is a reflection across the x-axis.



d. Sample answer: no; These functions are reflections along $y = -x$, which indicates that they are not inverses

58. **CHALLENGE** Solve $4^x - 2^{x+1} = 15$ for x.

SOLUTION:

$$4^x - 2^{x+1} = 15$$
$$\left(2^2\right)^x - 2^{x+1} = 15$$
$$\left(2^x\right)^2 - 2^x \cdot 2 = 15$$

Let $2^x = y$

$$y^2 - 2y = 15$$
$$y^2 - 2y - 15 = 0$$
$$(y-5)(y+2) = 0$$

By the Zero Product Property:

$$\begin{array}{lll} y-5=0 & \text{or} & y+2=0 \\ y=5 & \text{or} & y=-2 \\ 2^x=5 & \text{or} & 2^x=-2 \\ x\log 2=\log 5 & \text{or} & x\log 2=\log(-2) \end{array}$$

Logarithms are not defined for negative values.

Therefore, $x = \dfrac{\log 5}{\log 2} \approx 2.3219$

59. **PROOF** Prove $\ln ab = \ln a + \ln b$ for natural logarithms.

SOLUTION:

Let $p = \ln a$ and $q = \ln b$.

That means that $e^p = a$ and $e^q = b$.

$$ab = e^p \times e^q$$
$$ab = e^{p+q}$$
$$\ln(ab) = (p+q)$$
$$\ln(ab) = \ln a + \ln b$$

60. **REASONING** Determine whether $x > \ln x$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

SOLUTION:

Sample answer: Always; the graph of $y = x$ is always greater than the graph of $y = \ln x$ and the graphs never intersect.

61. **OPEN ENDED** Express the value 3 using e^x and the natural log.

SOLUTION:

Sample answer: $e^{\ln 3}$

62. **WRITING IN MATH** Explain how the natural log can be used to solve a natural base exponential function.

SOLUTION:

Sample answer: The natural log and natural base are inverse functions, so taking the natural log of a natural base will undo the natural base and make the problem easier to solve.

63. Given the function $y = 2.34x + 11.33$, which statement best describes the effect of moving the graph down two units?

A The x-intercept decreases.
B The y-intercept decreases.
C The x-intercept remains the same.
D The y-intercept remains the same.

SOLUTION:

The y-intercept decreases if the graph moves down two units.
Therefore, option B is the correct answer.

64. **GRIDDED RESPONSE** Aidan sells wooden picture frames over the Internet. He purchases supplies for \$85 and pays \$19.95 for his website. If he charges \$15 for each frame, how many will he need to sell in order to make a profit of at least \$270.

SOLUTION:

Let x be the number of frames.
She will earn $15x$ for each necklace that she sells but will need to subtract from that her fixed costs of supplies (\$85) and website fee (\$19.95)

$$\begin{aligned}
270 &< 15x - 85 - 19.95 \\
270 &< 15x - 104.95 \\
374.95 &< 15x \\
24.997 &< x
\end{aligned}$$

Therefore, she needs to sell 25 frames to make a profit of at least \$270.

65. Solve $|2x - 5| = 17$.
F −6, −11
G −6, 11
H 6, −11
J 6, 11

SOLUTION:

$$
\begin{array}{lcl}
|2x-5| = 17 & & \\
2x - 5 = 17 & \text{and} & -(2x-5) = 17 \\
2x = 22 & \text{and} & -2x + 5 = 17 \\
x = 11 & \text{and} & -2x = 12 \\
x = 11 & \text{and} & x = -6
\end{array}
$$

The solutions are –6 and 11.
Therefore, option G is the correct answer.

66. A local pet store sells rabbit food. The cost of two 5-pound bags is $7.99. The total cost *c* of purchasing *n* bags can be found by—
A multiplying *n* by *c*.
B multiplying *n* by 5.
C multiplying *n* by the cost of 1 bag.
D dividing *n* by *c*.

SOLUTION:
The total cost *c* of purchasing *n* bags can be found by multiplying *n* by the cost of 1 bag.
Therefore, option C is the correct answer.

Solve each equation or inequality. Round to the nearest ten-thousandth

67. $2^x = 53$

SOLUTION:

$$
\begin{aligned}
2^x &= 53 \\
\ln 2^x &= \ln 53 \\
x \ln 2 &= \ln 53 \\
x &= \frac{\ln 53}{\ln 2} \\
&\approx 5.7279
\end{aligned}
$$

68. $2.3^{x^2} = 66.6$

SOLUTION:

$$\begin{aligned} 2.3^{x^2} &= 66.6 \\ \ln 2.3^{x^2} &= \ln 66.6 \\ x^2 \ln 2.3 &= \ln 66.6 \\ x^2 &= \frac{\ln 66.6}{\ln 2.3} \\ x &= \pm\sqrt{\frac{\ln 66.6}{\ln 2.3}} \\ &\approx \pm 2.2452 \end{aligned}$$

69. $3^{4x-7} < 4^{2x+3}$

SOLUTION:

$$\begin{aligned} 3^{4x-7} &< 4^{2x+3} \\ \ln 3^{4x-7} &< \ln 4^{2x+3} \\ (4x-7)\ln 3 &< (2x+3)\ln 4 \\ (4x-7)1.0986 &< (2x+3)1.3863 \\ 4.3944491x - 7.6902860 &< 2.7725887x + 4.1588831 \\ 1.6218604x &< 11.849169 \\ x &< 7.3059 \end{aligned}$$

The solution region is $\{x \mid x < 7.3059\}$.

70. $6^{3y} = 8^{y-1}$

SOLUTION:

$$\begin{aligned} 6^{3y} &= 8^{y-1} \\ \log 6^{3y} &= \log 8^{y-1} \\ 3y \log 6 &= (y-1)\log 8 \\ 3y\frac{\log 6}{\log 8} &= y-1 \\ 3y(0.8617) &= y-1 \\ 2.5851y &= y-1 \\ 1.5851y &= -1 \\ y &= -0.6309 \end{aligned}$$

The solution is –0.6309.

71. $12^{x-5} \geq 9.32$

SOLUTION:

$$\begin{aligned} 12^{x-5} &\geq 9.32 \\ \log 12^{x-5} &\geq \log 9.32 \\ (x-5)\log 12 &\geq \log 9.32 \\ x-5 &\geq 0.8983 \\ x &\geq 5.8983 \end{aligned}$$

The solution region is $\{x \mid x \geq 5.8983\}$.

72. $2.1^{x-5} = 9.32$

SOLUTION:

$$\begin{aligned} 2.1^{x-5} &= 9.32 \\ \log 2.1^{x-5} &= \log 9.32 \\ (x-5)\log 2.1 &= \log 9.32 \\ x-5 &= \frac{\log 9.32}{\log 2.1} \\ x &= \frac{\log 9.32}{\log 2.1} + 5 \\ &\approx 8.0086 \end{aligned}$$

The solution is 8.0086.

73. **SOUND** Use the formula $L = 10 \log_{10} R$, where L is the loudness of a sound and R is the sound's relative intensity. Suppose the sound of one alarm clock is 80 decibels. Find out how much louder 10 alarm clocks would be than one alarm clock.

SOLUTION:

Substitute 80 for L and solve for R.

$$\begin{aligned} 80 &= 10 \log_{10} R \\ 8 &= \log_{10} R \\ R &= 10^8 \end{aligned}$$

If 10 alarm clocks ring at a time, the relative velocity of the sound is 10×10^8.
Substitute 10×10^8 for R and solve for L.

$$\begin{aligned} L &= 10 \log_{10}\left(10 \times 10^8\right) \\ &= 10 \log_{10} 10^9 \\ &= 90 \end{aligned}$$

The loudness would be increased by 10 decibels.

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

74. $x^3 + 5x^2 + 8x + 4$; $x + 1$

SOLUTION:

Divide the polynomial $x^3 + 5x^2 + 8x + 4$ by $x + 1$.

$$
\begin{array}{r}
x^2 + 4x + 4 \\
x+1\overline{)x^3 + 5x^2 + 8x + 4} \\
\underline{(-)\ x^3 + x^2 \qquad\qquad} \\
4x^2 + 8x \quad \\
\underline{(-)4x^2 + 4x \quad} \\
4x + 4 \\
\underline{(-)\ 4x + 4} \\
0
\end{array}
$$

Factor the quotient $x^2 + 4x + 4$.

$$
\begin{aligned}
x^2 + 4x + 4 &= x^2 + 2x + 2x + 4 \\
&= x(x+2) + 2(x+2) \\
&= (x+2)(x+2)
\end{aligned}
$$

Therefore, the factors are $x + 2$ and $x + 2$.

75. $x^3 + 4x^2 + 7x + 6$; $x + 2$

SOLUTION:

Divide the polynomial $x^3 + 4x^2 + 7x + 6$ by $x + 2$.

$$
\begin{array}{r}
x^2 + 2x + 3 \\
x+2\overline{)x^3 + 4x^2 + 7x + 6} \\
\underline{(-)\ x^3 + 2x^2 \qquad\qquad} \\
2x^2 + 7x \quad \\
\underline{(-)2x^2 + 4x \quad} \\
3x + 6 \\
\underline{(-)\ 3x + 6} \\
0
\end{array}
$$

The quotient is a prime. So the factor is $x^2 + 2x + 3$.

76. **CRAFTS** Mrs. Hall is selling crocheted items. She sells large afghans for $60, baby blankets for $40, doilies for $25, and pot holders for $5. She takes the following number of items to the fair: 12 afghans, 25 baby blankets, 45 doilies, and 50 pot holders.

a. Write an inventory matrix for the number of each item and a cost matrix for the price of each item.

b. Suppose Mrs. Hall sells all of the items. Find her total income as a matrix.

SOLUTION:

a. $\begin{bmatrix} 12 & 25 & 45 & 50 \end{bmatrix}$

$$\begin{bmatrix} 60 \\ 40 \\ 25 \\ 5 \end{bmatrix}$$

b. Multiply the matrixes.

$$\begin{aligned} \begin{bmatrix} 12 & 25 & 45 & 50 \end{bmatrix} \cdot \begin{bmatrix} 60 \\ 40 \\ 25 \\ 5 \end{bmatrix} &= \begin{bmatrix} 720 + 1000 + 1155 + 250 \end{bmatrix} \\ &= \begin{bmatrix} 3095 \end{bmatrix} \end{aligned}$$

Solve each equation.

77. $2^{3x + 5} = 128$

SOLUTION:

$$\begin{aligned} 2^{3x+5} &= 128 \\ \log 2^{3x+5} &= \log 128 \\ (3x+5)\log 2 &= \log 128 \\ 3x + 5 &= \frac{\log 128}{\log 2} \\ 3x &= \frac{\log 128}{\log 2} - 5 \\ x &= \frac{\frac{\log 128}{\log 2} - 5}{3} \\ &= \frac{2}{3} \end{aligned}$$

The solution is $\frac{2}{3}$.

78. $5^{n-3} = \frac{1}{25}$

SOLUTION:

$$\begin{aligned}
5^{n-3} &= \frac{1}{25} \\
\log 5^{n-3} &= \log \frac{1}{25} \\
(n-3)\log 5 &= \log 5^{-2} \\
(n-3)\log 5 &= -2\log 5 \\
n-3 &= -2 \\
n &= 1
\end{aligned}$$

The solution is 1.

79. $\left(\frac{1}{9}\right)^m = 81^{m+4}$

SOLUTION:

$$\begin{aligned}
\left(\frac{1}{9}\right)^m &= 81^{m+4} \\
\log\left(\frac{1}{9}\right)^m &= \log 81^{m+4} \\
\log \frac{1}{9^m} &= \log\left(9^2\right)^{m+4} \\
\log 9^{-m} &= \log 9^{2m+8} \\
-m\log 9 &= (2m+8)\log 9 \\
-m &= 2m+8 \\
m &= -\frac{8}{3}
\end{aligned}$$

The solution is $-\frac{8}{3}$.

80. $\left(\frac{1}{7}\right)^{y-3} = 343$

SOLUTION:

$$\begin{aligned} \left(\frac{1}{7}\right)^{y-3} &= 343 \\ \log\left(\frac{1}{7}\right)^{y-3} &= \log 343 \\ \log \frac{1}{7^{y-3}} &= \log 7^3 \\ \log 7^{-(y-3)} &= \log 7^3 \\ -(y-3)\log 7 &= 3\log 7 \\ -(y-3) &= 3 \\ y &= 0 \end{aligned}$$

The solution is 0.

81. $10^{x-1} = 100^{2x-3}$

SOLUTION:

$$\begin{aligned} 10^{x-1} &= 100^{2x-3} \\ 10^{x-1} &= \left(10^2\right)^{2x-3} \\ 10^{x-1} &= 10^{4x-6} \\ \log 10^{x-1} &= \log 10^{4x-6} \\ (x-1)\log 10 &= (4x-6)\log 10 \\ x-1 &= 4x-6 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

The solution is $\frac{5}{3}$.

82. $36^{2p} = 216^{p-1}$

SOLUTION:

$$\begin{aligned} 36^{2p} &= 216^{p-1} \\ \left(6^2\right)^{2p} &= \left(6^3\right)^{p-1} \\ 6^{4p} &= 6^{3p-3} \\ 4p &= 3p-3 \\ p &= -3 \end{aligned}$$

The solution is 3.